Turbulent Surface Flow around a Yawed Cone

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Nomenclature

F	$=1-\exp(-y\sqrt{\tau_w\bar{\rho}_w}/26\bar{\mu}_w)$, Van Driest's wall damping factor
i	= angle of attack
K	= coefficient in turbulent viscosity laws, 0.400
ℓ	= mixing length
M	= Mach number
Re	= local Reynolds number per unit length, m^{-1}
T	= temperature
$\bar{u},\bar{v},\bar{w}$	= mean velocity components along the x,y, and ϕ directions
x	= coordinate along generators of cone
y	= coordinate normal to surface
α_p	= streamline direction at the wall
γ_K	= intermittency factor
δ	= boundary-layer thickness, $\bar{u}/\bar{u}_e=0.99$
δ^*_{3D}	= Clauser's thickness
λ	= coefficient in turbulent viscosity laws, 0.168
θ_c	= cone half-angle
$\bar{\mu},\bar{\mu}_t$	= molecular turbulent viscosity coefficient
$\bar{\rho}$	= mean mass density
ϕ	= azimuthal angle
τ	= total shear stress

Subscripts

e	= external flow condition
w	= wall condition
∞	= freestream condition

Abstract

THE study of the three-dimensional boundary layer around a cone placed at angle of attack in a supersonic stream is carried out from the experimental and theoretical points of view. The results concern the limiting flow at the wall of a 7.5-deg half-angle cone at $M=5$. The limiting streamline directions α_p are measured from the surface oil flow patterns obtained for relative incidence values ranging up to $i/\theta_c=1$. The calculations are conducted in the attached region of the boundary layer. The turbulent viscosity concept and the y/δ similarity assumption are used to solve the Prandtl boundary-layer equations. The predictions obtained with four various laws are found to be in good agreement with the experimental data except in the neighborhood of the boundary-layer separation.

Contents

The present work is devoted to the problem of the turbulent three-dimensional boundary layer developed around bodies at supersonic speeds from both the experimental and theoretical points of view. The approach is relatively simple in the case of the supersonic flow over a yawed cone where the boundary-layer separation is due to an adverse pressure gradient acting only in the azimuthal direction.

The theoretical investigation of the flow near the wall is undertaken by using the Prandtl boundary-layer equations (zero approximation). In the present case, the turbulent terms destroy the similarity along each generator. But the independent variables number can be reduced to only two, as in the laminar case, by using local similarity equations.[1] With regard to the turbulent terms, the concept of isotropic eddy viscosity has been used under the form of the two-layer models as proposed by Smith and Cebeci[2]:

For inner region

$$\bar{\mu}_t=\bar{\rho}(Ky)^2F^2\left[\left(\frac{\partial\bar{u}}{\partial y}\right)^2+\left(\frac{\partial\bar{w}}{\partial y}\right)^2\right]^{1/2}$$

For outer region

$$\bar{\mu}_t=\bar{\rho}\lambda[\bar{u}_e^2+\bar{w}_e^2]^{1/2}\delta^*_{3D}\gamma_K$$

The laminar Prandtl number is obtained from National Bureau of Standards tables, whereas the turbulent Prandtl number is taken constant and equal to 0.89. The laminar viscosity is given by Sutherland's law.

The experimental investigation has been carried out in the hypersonic wind tunnel H_1 of the Institut de Mécanique de Fluides of Marseilles. This wind tunnel is of intermittent blowdown to vacuum type. The nozzle is axisymmetric and permits one to obtain a Mach number of 5 in the cylindrical circular test section, of 0.2 m diam. The tests are performed at the following stagnation conditions to get a turbulent boundary layer along the major cone length: $-p_{0\infty}=12.55\times10^5$ Pa, $T_{0\infty}=350$ K, and $Re_\infty=281\times10^7$/m.

The model is a 7.5-deg half-angle circular cone of 280 mm length. It is sting-mounted on a support, held from a side wall with an automatic gear for setting the model at incidence about 1 s after the beginning of the run. The basic type of experiments consists of thin oil dots spreading on the cone surface. The oil dots are applied in annular circles regularly located along the cone length and separated by about 20 mm. During the run, the oil is carried along the cone surface under the combined effects of skin friction and capillarity, and the traces obtained in this way are assumed to provide the limiting streamline direction α_p at the wall. The motion of a thin oil sheet on a surface under the influence of a boundary layer has been investigated by Squire.[3] The results of this study show the accuracy of this type of measurements, but they exhibit also that the separation line obtained by this means is found to be upstream of the true separation line. The streamline directions are recorded by developing the cone surface on an absorbing paper after the run. In order to make the measurements more accurate, the α_p value at a point located by the abscissa x and the azimuthal angle ϕ is taken equal to the average of the numerous values measured within the area limited by $x\pm1.5\times10^{-2}$ m and $\phi\pm5$ deg. The pattern of the experimental and theoretical distributions of α_p is strongly

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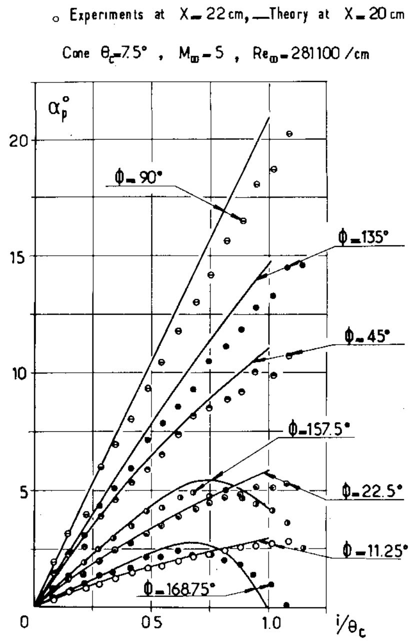

Fig. 1 Incidence angle effect on α_p.

dependent of the incidence and is shown in Fig. 1. The direction α_p is presented for different given azimuthal angles.

The external flow conditions are given by tabulated data of Jones.[4] Smith's eddy viscosity model without the intermittency factor has been chosen for the present comparisons because it permitted a better prediction than the others, as shown by many comparisons made in our laboratory. The agreement between theory and experiments is good enough, as shown on the figure, and, in particular, the evolution of the variable α_p with incidence appears to be well predicted. In the leeward region, the agreement also remains good when the evolution of α_p is no more monotonous; the location and the value of the maximum (for $\phi = 157.5$ and 168.75 deg) are predicted quite well.

From the comparisons that have been made from turbulent as well as from laminar conditions, the following remarks are made:

1) For small relative incidences ($i/\theta_c < 0.4$), the theoretical and experimental results are found to be in good agreement except in the region neighboring the 90-deg azimuth, where the calculated maximum value of α_p is slightly greater than the experimental one.

2) With increasing incidence ($i/\theta_c > 0.4$), the agreement remains similarly good except near the leeward half-plane, where the numerical solutions have a singular behavior that has been studied previously in the laminar case.[5] The parabolic Prandtl equations do not permit the boundary conditions to be satisfied in the leeward symmetry half-plane. Thus it would be necessary to maintain the elliptic character of the full Navier-Stokes equations in considering the azimuthal diffusion. The results so obtained by Roux and Forestier[6] are shown in the full paper for the laminar case.

3) When the incidence begins to be high ($i/\theta_c < 0.8$), the calculations lead to a separation of the laminar boundary layer, whereas such a separation is obtained experimentally only for $i/\theta_c \sim 1$. But one can note that the influence of viscous interaction is probably much more sensitive for the incidence values close to the incidence of separation when the pressure gradient just reaches the critical value, permitting the boundary-layer separation.

4) For a larger relative incidence, equal to unity, the predicted separation azimuth is notably smaller than the experimental one in the laminar case and than the predicted one in the turbulent case, whereas in this case the experiments do not yet reveal separation. On the windward side, the agreement between experiments and theory is still good, with the same overprediction of theory.

The flowfield has been investigated experimentally up to a relative incidence of 2, but the comparisons between experiments and computed data are limited to a relative incidence of unity, on account of the missing data of theoretical external flow conditions for larger incidences. Based on the results of this investigation in both laminar and turbulent cases, the following conclusions may be drawn:

1) On the windward side, the computed limiting flow at the wall is, at the measuring accuracy, in good agreement with experiments.

2) For moderate relative incidences, approximately between 0.3 and 0.7, the locally similar solutions of Prandtl's boundary-layer equations exhibit a singular behavior when $\phi \to 180$ deg. More complete equations including the terms of azimuthal diffusion have been tested in the laminar case and have permitted the boundary conditions to be satisfied in the leeward symmetry plane.

3) For high relative incidences close to unity, where the separation conditions are approached, the predicted values and the experimental data become substantially different. Then this disagreement concerns also the location of the separation line. It is, however, not possible to comment on the failure of this approach to predict the separation line for the aforementioned limitation of calculation for large incidence.

4) Because the variable α_p does not seem to be sensitive enough to test the validity of the eddy viscosity models, it may be desirable to extend this study to other variables across the boundary layer. Such a study is in progress at the Institut de Mécanique des Fluides of Marseilles.

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